Fast and Accurate Analysis of Interacting Fatigue Crack Growth with Boundary Cracklet Method

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Abstract

In this study, interacting crack growth in an infinite plate is analyzed with new, fast and accurate Boundary Cracklet Method (BCM) developed by Phoenix and Yavuz. An interior crack is under consideration to examine its propagation because of cyclic loading which is very common for aerospace, naval and civil engineering structures. BCM is used to determine the overall stress field as well as stress intensity factors for crack tips and singular wedges at crack kinks. BCM uses integral equations expressed in terms of unknown edge dislocation distributions along crack lines. These distributions derive from an accurate representation of the crack opening displacements using power series basis terms obtained from wedge eigenvalue analysis, which leads to both polynomial and non-polynomial power series. The process is to choose terms of the series and their exponents such that the tractions on the crack faces are virtually zero compared to the far field loading. Applying the method leads to a set of linear algebraic equations to solve for the unknown weighting coefficients for the power series basis terms to make no use of numerical integrations. A simple crack growth emanating from a triangular hole in an infinite plate is analyzed. The fatigue crack growth is assumed to follow Paris-Erdogan Law. The results are compared to those of other numerical methods. A parametric study is performed via graphs and tables to demonstrate the ability of BCM in analysis of fatigue crack growth.

Keywords

Edge Dislocations; Kinked Cracks; Edge Cracks; Opening Displacement Profile; Stress Intensity Factors; Fatigue Crack Growth; Paris Law

Introduction

Interaction and propagation of cracks is a significant concept in the design and analysis of aerospace, naval, and civil structures. Sharp corners at the holes bring about stress localizations that reduce strength and fatigue life.

In the current study, a simple straight crack growth emanating from a triangular hole in an infinite plate is analyzed with new, fast and accurate Boundary Cracklet Method (BCM) developed by Phoenix and Yavuz (Yavuz et al. (2006-2012)). An edge crack around the triangular hole is under consideration to observe its propagation. The fatigue crack growth is assumed to follow Paris-Erdogan Law. Triangular hole is modeled by straight cracks. Thus, they form an array of cracks within the structure. The study is performed via graphs and tables to demonstrate the ability of BCM in analysis of fatigue crack growth.

Introductory Remarks on Boundary Cracklet Method (BCM)

BCM is very useful to determine the overall stress field as well as stress intensity factors for crack tips and singular wedges at crack kinks. BCM uses integral equations expressed in terms of unknown edge dislocation distributions along crack lines. These distributions derive from an accurate representation of the crack opening displacements using power series basis terms obtained from wedge eigenvalue analysis, which leads to both polynomial and non-polynomial power series. The process is to choose terms of the series and their exponents such that the tractions on the crack faces are virtually zero compared to the far field loading. Applying the method leads to a set of

linear algebraic equations to solve for the unknown weighting coefficients for the power series basis terms to make no use of numerical integrations unlike in other methods. That’s why solution takes just a few seconds on a PC.

In the application, all boundaries and internal cracks are represented by cracklets. There are three types of cracklets (C1, C2 and C3) which have their own typical opening profile approximation functions. In the formulation, b is the opening profile approximation function, P’s are polynomial subfunction family, W’s are wedge subfunction family formed by rational powers of the position (t) in each cracklet. These rational powers are eigenvalues (ϱ) calculated from Williams Wedge Analysis. Depending on the wedge angle, there may be only one (in normal mode, mode I) or two (in both normal and tangential directions, mode I and mode II) eigenvalues. These eigenvalues are approximated by a ratio to evaluate all integrals analytically which makes the algorithm fast. These subfunctions are given for two modes of crack openings: i=1 for mode I, i=2 for mode II. All details of these opening profile functions and analytical and computational aspects of the method can be found in Yavuz et al (2006).

These three typical cracklets and corresponding approximation functions of opening displacement profiles are shown in Fig. 1. Cracklet type C1 is a cracklet with tip ends: A single straight crack is represented by this type of cracklet. Here only two wedge subfunctions with the power of (½) are used for each tip end points.

$$b_j^{(i)}(t) = W_{\frac{1}{2}j}^{A(i)}(t/a_i) + W_{\frac{1}{2}j}^{B(i)}(1-t/a_i) \qquad (1)$$

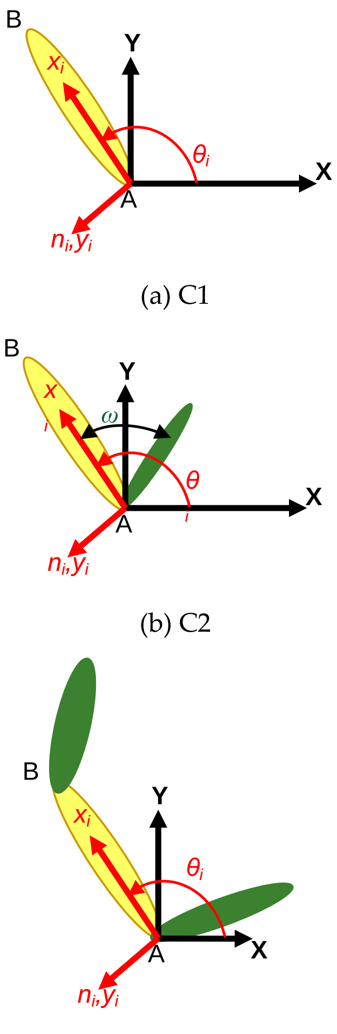
Cracklet type C2 is a cracklet with one wedge end and one tip end: A V-crack has two this type of cracklets. At tip end point B, the wedge eigenvalue is (½) so we have only one wedge subfunction family and for the other wedge end point A, there are one polynomial and one or two wedge subfunction families as C2.

$$b_j^{(i)}(t) = P_j^{A(i)}(t/a_i) + W_{\rho_1^A j}^{A(i)}(t/a_i)\left\langle + W_{\rho_2^A j}^{A(i)}(t/a_i)\right\rangle + W_{\frac{1}{2}j}^{B(i)}(1-t/a_i) \qquad (2)$$

Cracklet type C3 is a cracklet with only wedge ends: A kinked crack is represented by three cracklets and the yellow one is this type of cracklet. The opening profile approximation (b) may have from three to five subfunction families depending on the wedge angle at end points A and B.

$$b_j^{(i)}(t) = P_j^{AB(i)}(t/a_1) + W_{\rho_1^A j}^{A(i)}(t/a_1)\left\langle + W_{\rho_2^A j}^{A(i)}(t/a_1)\right\rangle + W_{\rho_1^B j}^{B(i)}(1-t/a_1)\left\langle + W_{\rho_2^B j}^{B(i)}(1-t/a_1)\right\rangle \qquad (3)$$

Even problems with curved boundaries and cracks can be solved by using small cracklets representing curved shapes. Since a small C2 type cracklet is added to each present crack's end to capture fatigue crack propagation after specific number of cyclic loads, all calculated SIFs at the end points become SIFs of the whole propagated crack configuration.



(a) C1

(b) C2

(c) C3

FIG. 1 CRACLET TYPES

Crack Growth and Propagation Model

Fatigue life calculations and crack growth behaviour are determined by Paris-Erdogan Law (Paris and Erdogan (1963)). Paris-Erdogan Law can be expressed by the formula given below,

$$\frac{dc}{dN} = C(\Delta K)^m \qquad (4)$$

$$\Delta K = K_{\max} - K_{\min} \qquad (5)$$

where dc/dN represents crack growth rate, ΔK is stress intensity range, C and m are material constants.

Constant amplitude cyclic loading with σ_{min}=0 is considered. Loading condition is illustrated in Fig. 2. Here, R is the stress ratio, σ_m is the midrange stress, and σ_a is the stress amplitude.

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \tag{6}$$

$$\sigma_a = \sigma_m = \sigma_{\max}/2 \tag{7}$$

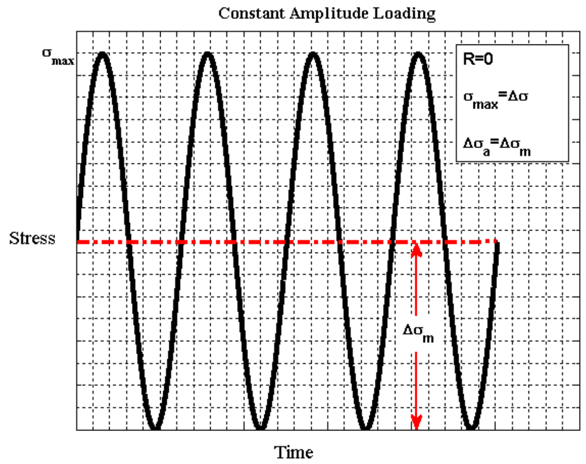


FIG. 2 CONSTANT AMPLITUDE LOADING WITH R=0 AND σ_{min}=0

Crack propagation and fatigue life estimation analysis is performed for the material parameters given in Table 1 (Yan (2006)).

TABLE 1 MATERIAL PARAMETERS

Material Parameters	Definition	Values
G	Shear Modulus [MPa]	26918.64
N	Poisson's Ratio	0.321
K_{IC}	Fracture toughness [MPa.mm1/2]	1137.96
ΔK_{TH}	SIF threshold [MPa.mm1/2]	0
C	Material constant [mm/cycle]	1.039×10^{-10}
m	Material constant	2.7438

Two different stress ranges ($\Delta\sigma$) are considered for the fatigue life and crack propagation analysis. Infinite plate is subjected to uniaxial tension in plane stress conditions. Initial crack orientations are taken as 0^0, 30^0, 45^0, and 60^0, respectively. For a 0^0 oriented crack subjected to uniaxial tension, Mode I condition is dominant. Since the orientations of the cracks that form triangular hole are different from 0^0, Mode II effects must be included. It is assumed that the fatigue crack growth is governed by the formula given below (Sih and Barthelemy (1980)),

$$\frac{dc}{dN} = B\left(\Delta K_{eff}\right)^s \tag{8}$$

where

$$\Delta K_{eff} = \frac{1}{2}\cos\frac{\theta_0}{2}\left[\Delta K_I(1+\cos\theta_0) - 3\Delta K_{II}\sin\theta_0\right] \tag{9a}$$

$$B=C,\ s=m \tag{9b}$$

where K_{eff} is the effective stress intensity factor, θ_0 is the crack growth angle, B and s are material constants. Crack propagation is performed by setting a constant crack propagation length for each step. Crack propagation angle is determined by considering the direction of the maximum tangential stress around the crack tip (Liang et al. (2003)).

$$c_{i+1} = c_i + \Delta c_i \quad \left(\Delta c_i : \text{constant}\right) \left(i = 1, 2, 3, \ldots, J\right) \tag{10}$$

$$\left(\theta_0\right)_i = 2\tan^{-1}\left(\frac{-2\left(\frac{\Delta K_{II}}{\Delta K_I}\right)_i}{1+\sqrt{1+8\left(\frac{\Delta K_{II}}{\Delta K_I}\right)_i^2}}\right) \tag{11}$$

Fatigue life calculation is followed by the formula given below,

$$N_i = \frac{c_i}{B\left(\Delta K_{eff}\right)_i} \quad \left(i = 1, 2, 3, \ldots, J\right) \tag{12}$$

Case Study (An Infinite Plate with a Triangular Hole Subjected to Uniaxial Tension)

The loading conditions and the geometry of the problem are shown in Fig. 3.

The stress intensity factors are compared with the results found in the literature to show the accuracy of BCM (Table 2). The results are found to be in a good agreement with existing solutions in studies Murakami (1987) and Yan (2005). The crack propagation is analyzed for four different initial crack orientations and two different stress ranges. The parameters are given in Table 3. The shape of the crack emanating from a triangular hole and its propagation is shown in Fig. 4. As expected, the crack becomes perpendicular to the applied load direction regardless of its initial orientation. The effective stress intensity factor versus crack length is given in Fig. 5 and Fig. 6. The higher stress amplitudes cause the higher stress intensity factors. The stress intensity factors are found to be higher for the lower values of initial crack

orientations. In the case of lower values of initial crack orientations, it is found that the number of cycles to reach a given crack size is lower compared to the one for the higher values of initial crack orientations (Fig. 7).

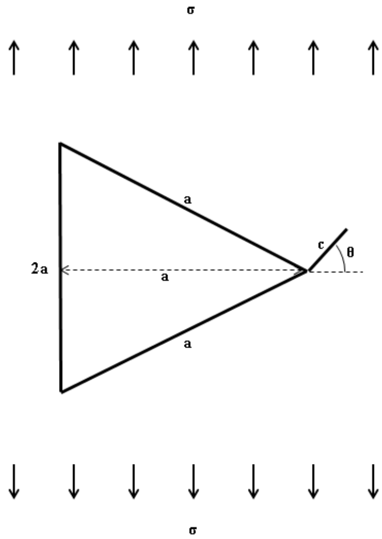


FIG. 3 AN INFINITE PLATE WITH A TRIANGULAR HOLE SUBJECTED TO UNIAXIAL TENSION

TABLE 2 COMPARISON OF THE STRESS INTENSITY FACTORS FOR A TRIANGULAR HOLE WITH CRACK (Θ=0°)

	Murakami (1987)	**Yan (2005)**	**BCM (current)**
c/a	$K_I/(\pi c)^{0.5}$	$K_I/(\pi c)^{0.5}$	$K_I/(\pi c)^{0.5}$
0.05	-	3.90	3.9148
0.1	2.76	2.8348	2.8414
0.2	2.064	2.0650	2.0686
0.4	1.525	1.5238	1.5266
0.6	1.297	1.2959	1.2975
0.8	1.168	1.1671	1.1688
1.0	1.085	1.0786	1.0863

TABLE 3 PARAMETERS OF THE CASE

	Case	**Definition**
$\Delta\sigma$	150.42, 300.83	Stress ranges [MPa]
Θ	0°, 30°, 45°, 60°	Orientation of the initial crack
c_i/a	0.2	Ratio of the length of the initial crack to the edge length (i=0)
$\Delta c/c_i$	0.1	Ratio of the crack length increment to the initial crack length (i=0)

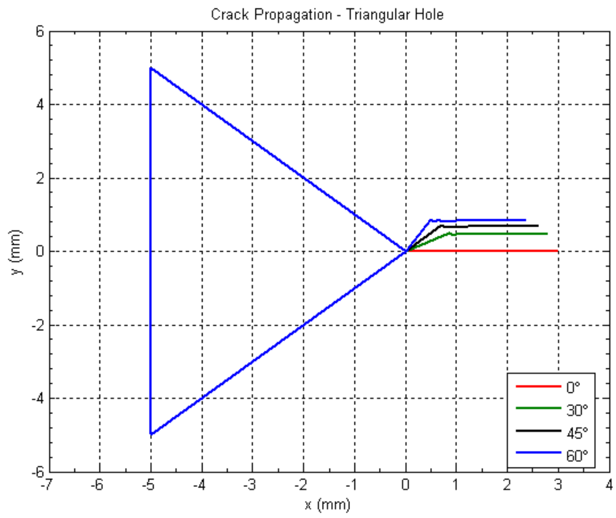


FIG. 4 CRACK PROPAGATION GRAPH

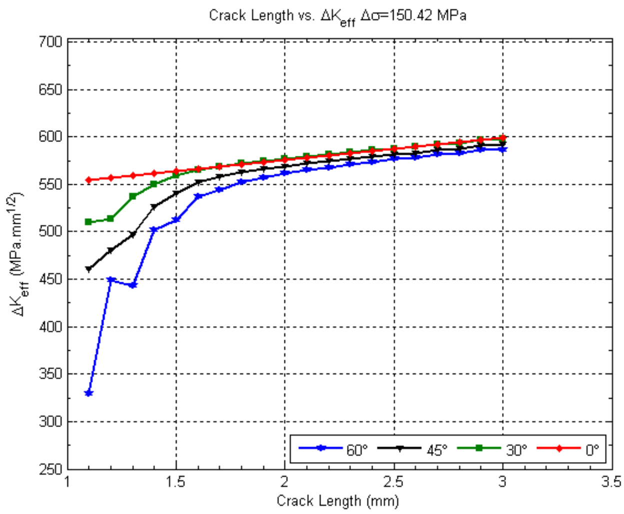


FIG. 5 CRACK LENGTH VS ΔK_{eff} ($\Delta\sigma$=150.42 MPa)

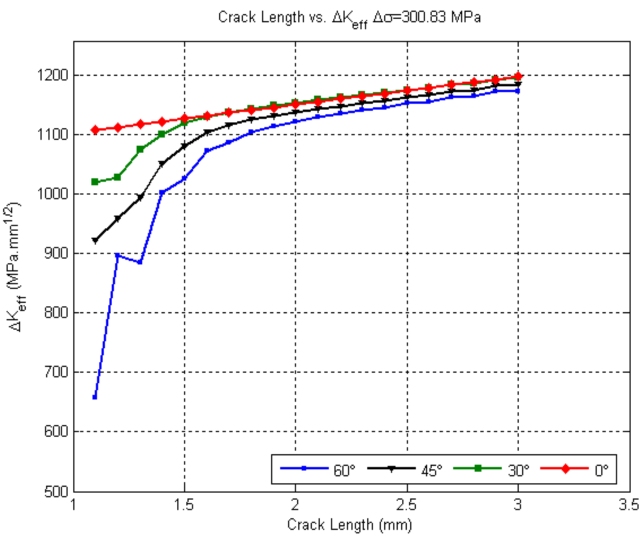


FIG. 6 CRACK LENGTH VS ΔK_{eff} ($\Delta\sigma$=300.83 MPa)

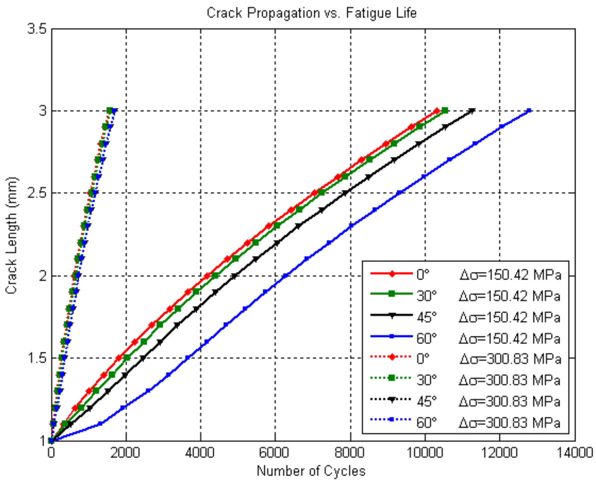


FIG. 7 CRACK LENGTH VS NUMBER OF CYCLES

Conclusion

The Boundary Cracklet Method is used to anticipate the stress intensity factors for the cracks emanating from a triangular hole during the cyclic loading. The stress intensity factors calculated for several initial crack orientations are found to be in an agreement with the ones found in the literature. The fatigue crack growth is assumed to follow Paris-Erdogan Law. The results are obtained for the propagation of cracks emanating from a triangular hole. As the number of cycles increases, the crack becomes perpendicular to the applied load direction regardless of its initial orientation as expected. Also, the higher stress amplitudes cause the higher stress intensity factors. The stress intensity factors are found to be higher for the lower values of initial crack orientations. In the case of lower values of initial crack orientations, it is found that the number of cycles to reach a given crack size is lower compared to the one for the higher values of initial crack orientations. The method presented here predicates the crack propagation for the cracks emanating from a triangular hole very well.

REFERENCES

Liang, J., R. Huang, J.H. Prévost, and Z. Suo. “Evolving crack patterns in thin films with the extended finite element method.” International Journal of Solids and Structures 40 (2003): 2343-2354.

Murakami, Y. Stress Intensity Factors Handbook. NY: Pergamon Press, 1987.

Paris, P., and F. Erdogan. “A critical analysis of crack propagation laws”, Journal of Basic Engineering, Transactions of the American Society of Mechanical Engineers (1963): 528-534.

Sih, G.C., and B.M. Barthelemy. “Mixed mode fatigue crack growth prediction.” Engineering Fracture Mechanics 13 (1980): 439-51.

Yan, X. “Numerical analysis of a few complex crack problems with a boundary element method.” Engineering Failure Analysis. 13 (2006): 805-825.

Yan, X. “Stress intensity factors for cracks emanating from a triangular or square hole in an infinite plate by boundary elements.” Engineering Failure Analysis. 12 (2005): 362-375.

Yavuz, A. Kadir, A. Deniz Senalp, Halit S. Turkmen. “Fast and Accurate Fatigue Crack Growth Analysis with the Boundary Cracklet Method.” Paper 226 in B.H.V. Topping, Y. Tsompanakis, (Editors), Proceedings of the Thirteenth International Conference on Civil, Structural and Environmental Engineering Computing, Stirlingshire, UK : Civil-Comp Press, 2011.

Yavuz, A. Kadir, S. Leigh Phoenix. “Easy Handling of Complex Multiple Crack Interactions by Boundary Cracklet Method (BCM).” 50th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, May 4-7, Palm Springs, CA, AIAA Paper No. AIAA-2009-2627, 2009.

Yavuz, A. Kadir, S.Leigh Phoenix, A. Deniz Senalp, Halit S. Turkmen. “Interacting Fatigue Crack Growth Analysis with Boundary Cracklet Method (BCM).” Advanced Materials Research 445 (2012): 1017-1022.

Yavuz, A. Kadir, S. Leigh Phoenix, and Stephanie C. TerMaath. “An accurate and fast analysis for strongly interacting multiple crack configurations including kinked(V) and branched(Y) cracks.” International Journal of Solids and Structures 43 (2006): 6727-6750.

Yavuz, A. Kadir, S. Leigh Phoenix, and Stephanie C. TerMaath. “Multiple Crack Analysis in Finite Plates.” AIAA Journal 44/11 (2006): 2535-2541.